Oxford Cambridge and RSA

Accredited

AS Level Mathematics B (MEI)

H630/02 Pure Mathematics and Statistics

Sample Question Paper

Version 2

Date – Morning/Afternoon

Model Answers

Time allowed: 1 hour 30 minutes

You must have:
- Printed Answer Booklet

You may use:
- a scientific or graphical calculator

INSTRUCTIONS

- Use black ink. HB pencil may be used for graphs and diagrams only.
- Complete the boxes provided on the Printed Answer Booklet with your name, centre number and candidate number.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION

- The total number of marks for this paper is **70**.
- The marks for each question are shown in brackets **[ ]**.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is used. You should communicate your method with correct reasoning.
- The Printed Answer Booklet consists of **12** pages. The Question Paper consists of **12** pages.

Formulae AS level Mathematics B (MEI) (H630)

Binomial series

$(a+b)^n = a^n + {}^n\mathrm{C}_1\, a^{n-1}b + {}^n\mathrm{C}_2\, a^{n-2}b^2 + \ldots + {}^n\mathrm{C}_r\, a^{n-r}b^r + \ldots + b^n \qquad (n \in \mathbb{N})$,

where ${}^n\mathrm{C}_r = {}_n\mathrm{C}_r = \binom{n}{r} = \dfrac{n!}{r!(n-r)!}$

$(1+x)^n = 1 + nx + \dfrac{n(n-1)}{2!}x^2 + \ldots + \dfrac{n(n-1)\ldots(n-r+1)}{r!}x^r + \ldots \qquad (|x|<1,\ n \in \mathbb{R})$

Differentiation from first principles

$\mathrm{f}'(x) = \lim\limits_{h \to 0} \dfrac{\mathrm{f}(x+h) - \mathrm{f}(x)}{h}$

Sample variance

$s^2 = \dfrac{1}{n-1}S_{xx}$ where $S_{xx} = \sum (x_i - \bar{x})^2 = \sum x_i^2 - \dfrac{\left(\sum x_i\right)^2}{n} = \sum x_i^2 - n\bar{x}^2$

Standard deviation, $s = \sqrt{\text{variance}}$

The binomial distribution

If $X \sim \mathrm{B}(n,\ p)$ then $P(X = r) = {}^n\mathrm{C}_r\, p^r q^{n-r}$ where $q = 1 - p$

Mean of X is np

Kinematics

Motion in a straight line

$v = u + at$

$s = ut + \frac{1}{2}at^2$

$s = \frac{1}{2}(u+v)t$

$v^2 = u^2 + 2as$

$s = vt - \frac{1}{2}at^2$

Answer **all** the questions

1 Find $\int \left(x^2 + \frac{1}{x^2}\right) \mathrm{d}x$. **[3]**

1. $\int x^2 + \frac{1}{x^2} \, dx = \left[\frac{1}{3}x^3 - \frac{1}{x} + c \right]$

2 **(a)** Express $2\log_3 x + \log_3 a$ as a single logarithm. **[1]**

2. a) $2\log_3 x + \log_3 a = \log_3 x^2 + \log_3 a$
$= \log_3 ax^2$

(b) Given that $2\log_3 x + \log_3 a = 2$, express x in terms of a. **[3]**

b) $\log_3 ax^2 = 2$
$ax^2 = 3^2$
$x^2 = \frac{9}{a}$
$x = \frac{3}{\sqrt{a}}$

x cannot be negative because you cannot have a logarithm of a negative number

3 Show that the area of the region bounded by the curve $y = 3x^{-\frac{3}{2}}$, the lines $x = 1$, $x = 3$ and the x-axis is $6 - 2\sqrt{3}$. **[5]**

3. $\int_1^3 3x^{-\frac{3}{2}} \, dx = \left[-6x^{-\frac{1}{2}}\right]_1^3$

$= -6(3)^{-\frac{1}{2}} + 6(1)^{-\frac{1}{2}}$

$= \frac{-6}{\sqrt{3}} + 6$

$= \frac{-6 + 6\sqrt{3}}{\sqrt{3}}$

$= \frac{-6+6\sqrt{3}}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$

$= \frac{-6\sqrt{3}+6(3)}{3}$

$= -2\sqrt{3}+6$

4 There are four human blood groups; these are called O, A, B and AB. Each person has one of these blood groups. The table below shows the distribution of blood groups in a large country.

Blood group	Proportion of population
O	49%
A	38%
B	10%
AB	3%

Two people are selected at random from this country.

(a) Find the probability that at least one of these two people has blood group O. **[2]**

4a) P(at least one is O) = 1 − P(none are O)
$= 1 - 0.51^2$
$= 0.7399$

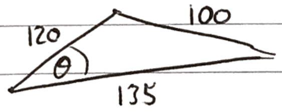
(b) Find the probability that each of these two people has a different blood group. **[3]**

b) = 1 − P(both O) − P(both A) − P(both B) − P(both AB)
$= 1 - 0.49^2 - 0.38^2 - 0.1^2 - 0.03^2$
$= 0.6046$

5 A triangular field has sides of length 100 m, 120 m and 135 m.

(a) Find the area of the field. **[5]**

5 a)



Cosine rule : $100^2 = 120^2 + 135^2 - 2(120)(135)\cos\theta$

$$\cos\theta = \frac{120^2 + 135^2 - 100^2}{2(120)(135)}$$

$\cos\theta = 0.698$

$\theta = 45.7$

Area $= \frac{1}{2}ab\sin C = \frac{1}{2}(120)(135)\sin 45.7$

$= 5798$

(b) Explain why it would not be reasonable to expect your answer in **(a)** to be accurate to the nearest square metre. **[1]**

b) The sides have been measured to the nearest meter so there are other possible areas

6 (a) The graph of $y = 3\sin^2\theta$ for $0° \le \theta \le 360°$ is shown in **Fig. 6**.

On the copy of **Fig. 6** in the Printed Answer Booklet, sketch the graph of $y = 2\cos\theta$ for $0° \le \theta \le 360°$. **[2]**

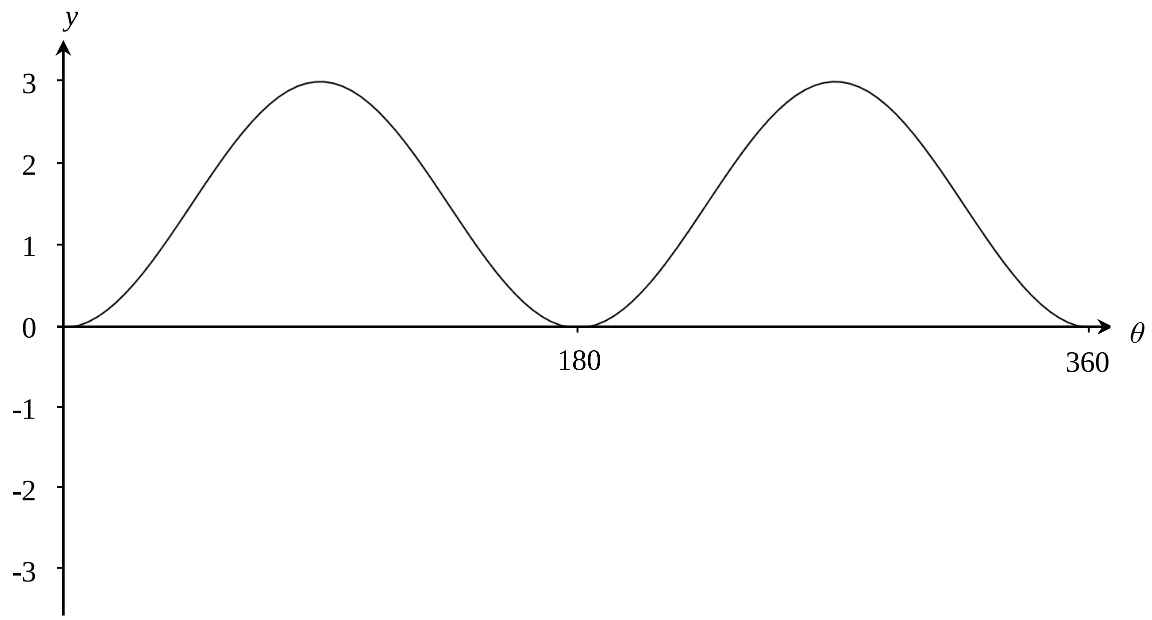
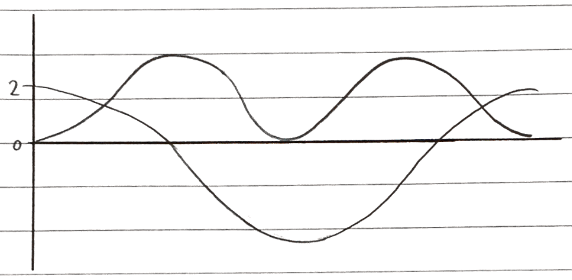


Fig. 6

6a)



(b) In this question you must show detailed reasoning.

Determine the values of θ, $0° \le \theta \le 360°$, for which the two graphs cross. **[6]**

b) $2\cos\theta = 3\sin^2\theta$

$2\cos\theta = 3(1 - \cos^2\theta)$

$3\cos^2\theta + 2\cos\theta - 3 = 0$

$\cos\theta = \dfrac{-2 \pm \sqrt{2^2 - 4(3)(-3)}}{2(3)}$

$\cos\theta = \frac{-2 \pm \sqrt{4 + 36}}{6}$

$\cos\theta = \frac{-1 \pm \sqrt{10}}{3}$

$\cos\theta = \frac{-1 - \sqrt{10}}{3} = -1.387$ no solutions

$\cos\theta = \frac{-1 + \sqrt{10}}{3} = 0.721$

7 A farmer has 200 apple trees. She is investigating the masses of the crops of apples from individual trees. She decides to select a sample of these trees and find the mass of the crop for each tree.

(a) Explain how she can select a random sample of 10 different trees from the 200 trees. **[2]**

The masses of the crops from the 10 trees, measured in kg, are recorded as follows.

23.5 27.4 26.2 29.0 25.1 27.4 26.2 28.3 38.1 24.9

7.a) Number each tree from 1 to 200
Choose 10 random numbers and use the corresponding trees

(b) For these data find
- the mean,
- the sample standard deviation. **[2]**

b) $\Sigma m = 276.1$

mean $= \frac{276.1}{10} = 27.61$ kg

sd $= 4.04$ kg

(c) Show that there is one outlier at the upper end of the data. How should the farmer decide whether to use this outlier in any further analysis of the data? **[3]**

c) upper limit = 27.61 + 2(4.04)
= 35.69

38.1 > 35.69 so it is an outlier

Check to see if this value is correct by checking to see if the apple did weigh this much. If it did, do not remove it from the data set

8 In an experiment, the temperature of a hot liquid is measured every minute.
The difference between the temperature of the hot liquid and room temperature is D °C at time t minutes.

Fig. 8 shows the experimental data.

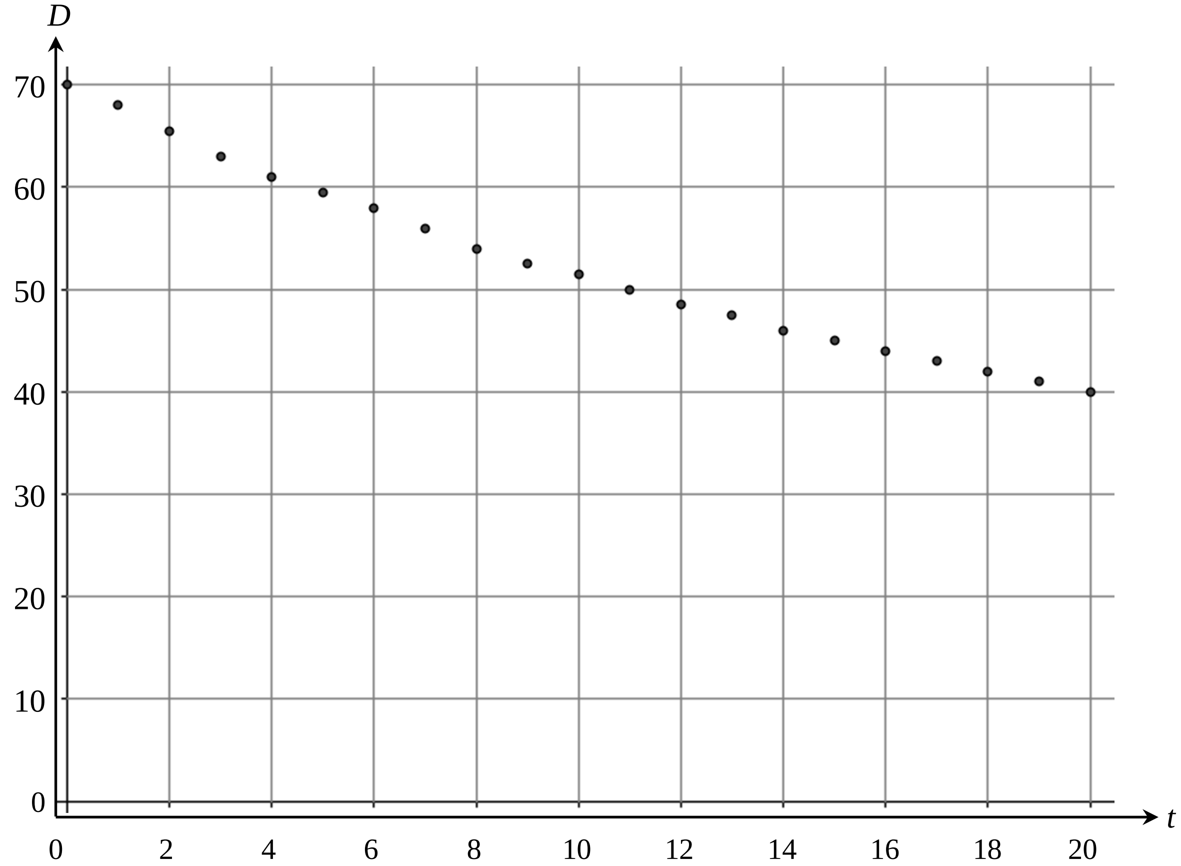


Fig. 8

It is thought that the model $D = 70e^{-0.03t}$ might fit the data.

(a) Write down the derivative of $e^{-0.03t}$. **[1]**

8 a) $-0.03e^{-0.03t}$

(b) Explain how you know that $70e^{-0.03t}$ is a decreasing function of t. **[1]**

b) $-0.03e^{-0.03t} < 0$ for all values of t so the gradient is always negative

(c) Calculate the value of $70e^{-0.03t}$ when

(i) $t = 0$, **[1]**

c) i) 70

(ii) $t = 20$. **[1]**

ii) $70e^{-0.03 \times 20} = 38.4$

(d) Using your answers to parts **(b)** and **(c)**, discuss how well the model $D = 70e^{-0.03t}$ fits the data. **[3]**

d) From the graph you can see that the data points decrease as t increases. This fits with b) where we found the model to be decreasing.
When t = 0, the graph has a value of D = 70 which fits exactly with the model
When t = 20, the graph has a value of D = 40 which is quite close to the prediction of the model
Therefore the model fits the data quite well

9 **Fig. 9.1** shows box and whisker diagrams which summarise the birth rates per 1000 people for all the countries in three of the regions as given in the pre-release data set.
The diagrams were drawn as part of an investigation comparing birth rates in different regions of the world.

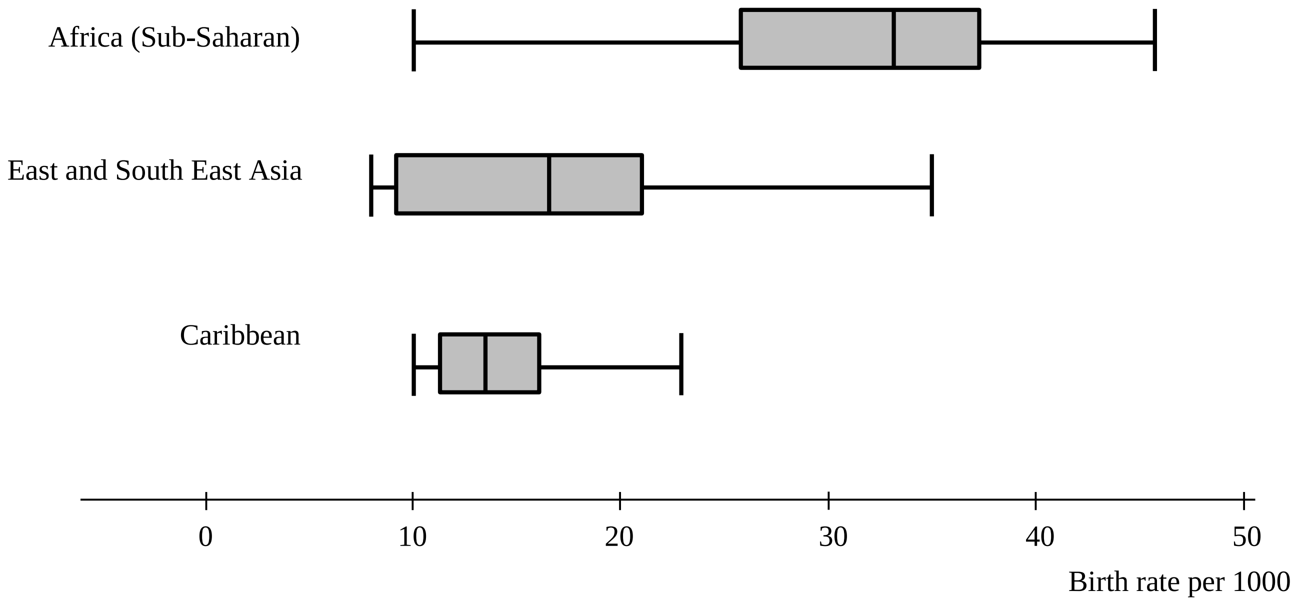


Fig. 9.1

(a) Discuss the distributions of birth rates in these regions of the world.
Make three different statements. You should refer to **both** information from the box and whisker diagrams **and** your knowledge of the large data set. **[3]**

9 a) There is a greater range of birth rates between the countries of sub-Saharan Africa than there is between countries of the Caribbean
This is because sub-Saharan Africa has a mix of richer and poorer economys, meaning there is a larger IQR.
The IQRs for sub-Saharan Africa and East and South East Asia are similar. However the African countries have a much larger ~~f~~ range which could be caused by outliers

(b) The birth rates for all the countries in Australasia are shown below.

Country	**Birth rate per 1000**
Australia	12.19
New Zealand	13.4
Papua New Guinea	24.89

(i) Explain why the calculation below is not a correct method for finding the birth rate per 1000 for Australasia as a whole.

$$\frac{12.19+13.4+24.89}{3}\approx 16.83$$ **[1]**

b) i. Does not take the populations into account

(ii) Without doing any calculations, explain whether the birth rate per 1000 for Australasia as a whole is higher or lower than 16.83. **[1]**

ii. Lower because Australia has the highest population so should have been given more weight. It has the lowest birth rate so would decrease the birth rate of Australasia.

The scatter diagram in **Fig. 9.2** shows birth rate per 1000 and physicians/1000 population for all the countries in the pre-release data set.

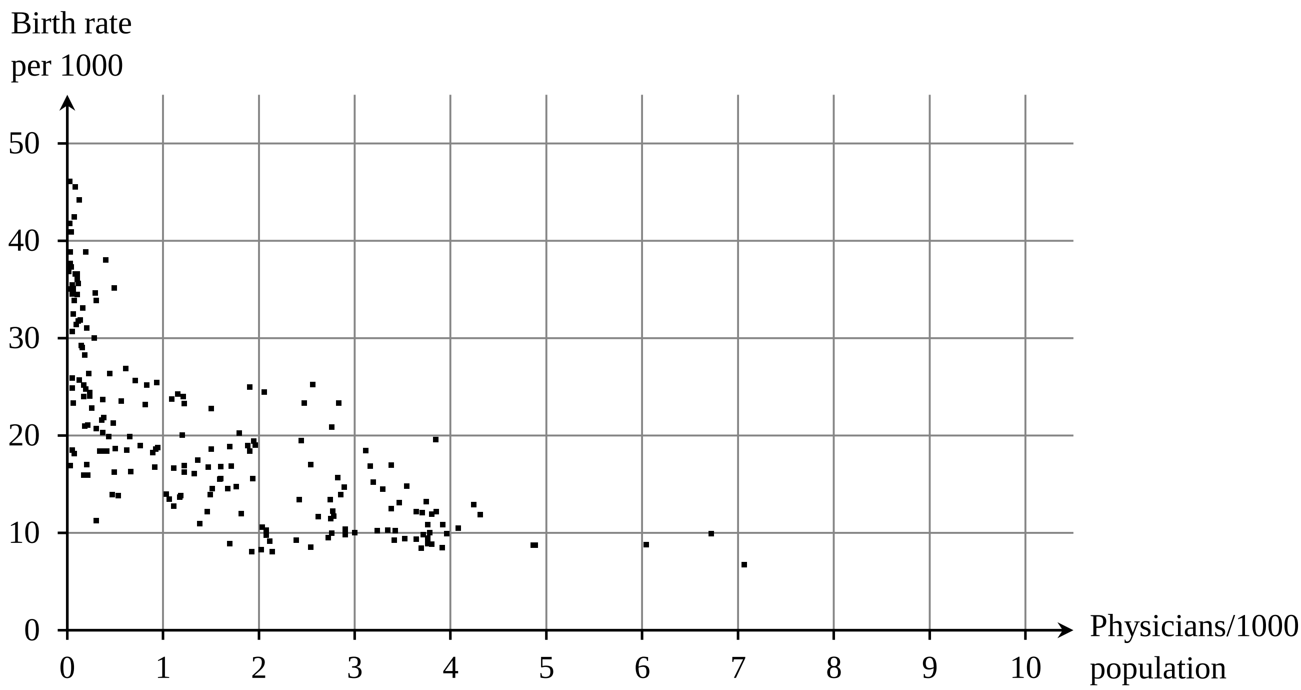


Fig. 9.2

(c) Describe the correlation in the scatter diagram. **[1]**

c) Negative correlation

(d) Discuss briefly whether the scatter diagram shows that high birth rates would be reduced by increasing the number of physicians in a country. **[1]**

d) Correlation does not always imply causation, so you cannot be certain

10 A company operates trains. The company claims that 92% of its trains arrive on time. You should assume that in a random sample of trains, they arrive on time independently of each other.

(a) Assuming that 92% of the company's trains arrive on time, find the probability that in a random sample of 30 trains operated by this company

(i) exactly 28 trains arrive on time, **[2]**

10 a) let X be the number of trains that arrive on time

$X \sim B(30, 0.92)$

i) $P(X = 28) = \binom{30}{28} \times 0.92^{28} \times 0.08^{2}$

$= 0.2696$

(ii) more than 27 trains arrive on time. **[2]**

ii) $P(X > 27) = 1 - P(X \leq 27)$

$= 1 - 0.4346$

$= 0.5654$

A journalist believes that the percentage of trains operated by this company which arrive on time is lower than 92%.

(b) To investigate the journalist's belief a hypothesis test will be carried out at the 1% significance level. A random sample of 18 trains is selected.
For this hypothesis test,

- state the hypotheses,
- find the critical region. **[5]**

b) $H_0 : p = 0.92$ where p is the probability that

$H_1 : p < 0.92$ a train arrives on time

Under H_0, $X \sim B(18, 0.92)$

$P(X \leqslant 13) = 0.0116$

$P(X \leqslant 12) = 0.0021$

$0.0021 < 0.01$ so $X \leqslant 12$ is the critical region

11 **In this question you must show detailed reasoning.**

Fig. 11 shows the curve $y = f(x)$, where $f(x)$ is a cubic function. **Fig. 11** also shows the coordinates of the turning points and the points of intersection with the axes.

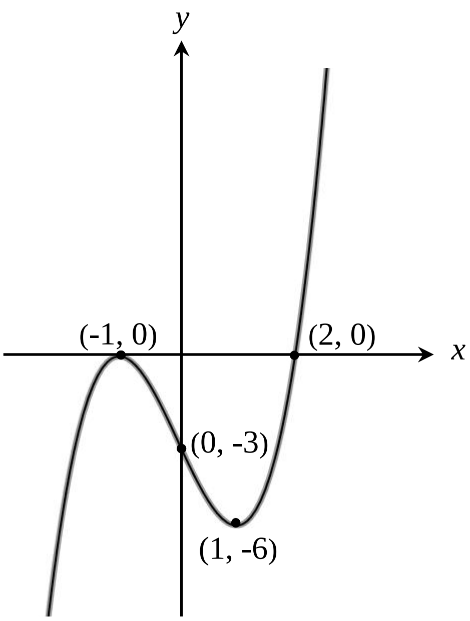


Fig. 11

Show that the tangent to $y = f(x)$ at $x = t$ is parallel to the tangent to $y = f(x)$ at $x = -t$ for all values of t. **[6]**

11 Equation of curve is $y = k(x+1)^2(x-2)$

Sub in $(0, -3)$ to find k

$-3 = k(1)^2(-2)$

$k = \frac{3}{2}$

$y = \frac{3}{2}(x+1)^2(x-2)$

$y = \frac{3}{2}(x^2+2x+1)(x-2)$

$y = \frac{3}{2}(x^3 - \cancel{2x^2} + \cancel{2x^2} - 4x + x - 2)$

$y = \frac{3}{2}x^3 - \frac{9}{2}x - 3$

$\frac{dy}{dx} = \frac{9}{2}x^2 - \frac{9}{2}$

At $x = t$, $\frac{dy}{dx} = \frac{9}{2}t^2 - \frac{9}{2}$

At $x = -t$, $\frac{dy}{dx} = \frac{9}{2}t^2 - \frac{9}{2}$

The two gradients are equal so the tangents are parallel

12 Given that $\arcsin x = \arccos y$, prove that $x^2 + y^2 = 1$. [Hint: Let $\arcsin x = \theta$] **[3]**

12 let $\arcsin x = \theta$

$x = \sin\theta$

$\arccos y = \arcsin x$

$\arccos y = \theta$

$y = \cos\theta$

$\sin^2\theta + \cos^2\theta = 1$

$x^2 + y^2 = 1$

END OF QUESTION PAPER